Mechanical Stresses Analysis of Thin Round Membranes in the Case of Large Deflections

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***Abstract*— An analysis of the methods for calculating the dependence of the deflection of multilayer round membranes on the applied overpressure was carried out. To compare theoretical calculations with experimental data, as an example, we considered a round membrane based on the structure Al/SiO2/Al with a total thickness of 2.5 µm. Using the COMSOL Multiphysics and ANSYS software packages, we calculated the stress distribution in the radial and tangential directions, the distribution of equivalent von Mises stresses over the membrane area, and also compared the obtained numerical values of mechanical stresses with the corresponding analytical estimates. We compared the deflection of the membrane depending on the applied overpressure obtained using software packages, analytical estimates and records obtained experimentally using an optical profilometer. It was shown that, when studying the deflection of membranes, which significantly exceeds the thickness of the membranes, to predict the deformations expected in the experiment, analytical relations obtained taking into account the presence of tensile forces, radial displacement, and elongation of the median plane should be used.**

Keywords*— mechanical strength; mechanical stress; membrane; MEMS; thin film; von Mises stress; finite-element numerical calculations

I. Introduction

At present, many studies [1—5] are devoted to the investigation of the dependence of the deflection of membranes on the applied excess pressure. Based on the obtained dependences, the researchers determine the mechanical characteristics of the structures — mechanical stresses and Young's modulus [1, 5—7], evaluate the strength and determine the maximum pressure that the structures can withstand [6, 7], and develop possibilities for handling mechanical stresses [1, 5]. The mechanical properties of membranes, which are a key functional element of MEMS (micro-electro-mechanical system) technology, often primarily determine the functionality of devices based on them, which emphasizes the relevance and significance of such studies for the successful implementation of developments in the field of micro- and nanoelectronics [8].

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II. Method for Analysis of the Deflection of Round Membranes

Flat membranes, which are round plates (wafers), fixed along the contour and loaded uniformly by pressure, were studied.

The method of analysis of membrane deflection is based on solving problems of the theory of elasticity. In this case, the following relation is often used for a plate with a closed contour [9—10]:

$$w = \frac{p}{64D}\left(a^2 - r^2\right)^2 \quad (1)$$

$$w_{\max} = \frac{pa^4}{64D} \quad (2)$$

where a – radius of the circular membrane, r – distance from the center of the membrane, w – displacement of the membrane at a distance r, p – applied overpressure, $D = \frac{Eh^3}{12\left(1-\mu^2\right)}$ – bending (cylindrical) hardness of the plate, h – thickness of the membrane, μ – Poisson's ratio, E – Young's modulus, w_{max} – largest value of displacement or deflection (in the center of the plate).

Relations for stresses in radial σ_r and tangential σ_t directions are also known and widely used [7, 10]:

$$\sigma_r = \frac{3p}{8h^2}\left(a^2\left(1+\mu\right) - r^2\left(3+\mu\right)\right) \quad (3)$$

$$\sigma_t = \frac{3p}{8h^2}\left(a^2\left(1+\mu\right) - r^2\left(1+3\mu\right)\right) \quad (4)$$

However, it should be considered that these ratios are obtained under the assumption of relatively small deflections of the wafer, i.e. in the case when the displacement of the membranes occurs mainly due to bending deformations.

If the value of deflection of the membrane significantly exceeds the thickness of the membrane, it is necessary to take into account the deformation (elongation) of the middle layer.

For this purpose, the Karman system of equations can be used, which are nonlinear partial differential equations and describe the elastic behavior of the wafer over the entire range of its deflections [11]. Since, with the exception of special cases, these equations are not exactly integrated in the general form, so to analyze the dependence of the deflection on the applied overpressure, approximate solutions of the Karman equation for a wafer (membrane) rigidly clamped along the contour are most often used [12]:

$$p = C_0 \frac{Eh^3}{a^4(1-\mu^2)} w_{max} + C_1 \frac{\sigma_0 h w_{max}}{a^2} + C_2 \frac{Eh}{a^4(1-\mu^2)} w_{max}^3 \quad (5)$$

where σ_0 – initial internal stress of the membrane. The values of the coefficients C_0, C_1, C_2 depend on the shape of the membrane and are usually refined during specific calculations using mathematical modeling methods.

Expressions of the dependence of the deflection and stresses on the applied overpressure obtained analytically are also known [9], made taking into account the presence of tensile forces, radial displacement and elongation of the median plane:

$$\sigma_r = \frac{Ew_{max}^2}{4a^2}\left(\frac{3-\mu}{1-\mu} - \left(\frac{r}{a}\right)^2\right) \quad (6)$$

$$\sigma_t = \frac{Ew_{max}^2}{4a^2}\left(\frac{3-\mu}{1-\mu} - 3\left(\frac{r}{a}\right)^2\right) \quad (7)$$

$$w_{max}^3 = \frac{pa^4h^3 3(1-\mu)}{Eh^4(7-\mu)} \quad (8)$$

For calculations with $\mu = 0.3$, it is recommended to use the expression [9]:

$$\frac{3,58 w_{max}^3}{h^3} = \frac{pa^4}{Eh^4} \quad (9)$$

A significant difference between representation (6-9) and representation (1-2) is the cubic dependence of the deflection of the membrane (in the case of large deflections), which is observed in the case when the membrane operates mainly with tension, in contrast to the case of a linear dependence when the membrane operates in the region of small displacements.

In this regard, expressions (6-9) are comparable with expression (5), in which the second term describes the linear region of the dependence of the deflection of the membrane on pressure, and the third term takes into account nonlinear deformations.

This study is devoted to the analysis of the dependence of the deflection on the applied overpressure, radial σ_R and tangential σ_T components of mechanical stresses, σ_μ von Mises stress [13], comparison of various data processing methods.

Presently in manufacturing practice, researchers often deal with membranes, the deflection of which significantly exceeds their thickness. For example, membranes that are made using silicon technology have a thickness of the order of units of micrometers or less, while the deflection of such membranes can reach 50 microns or more. However, for the analysis of the dependence of the deflection of membranes on the overpressure in the case of the structures noted above, relations (1) can be used [7], since they are convenient for fast qualitative calculations. Since such a representation can lead to errors in the calculations, the goal of this study is to estimate the error associated with such a representation.

III. Experiment. Sample Preparation

To compare the results of computations with experimental data, samples of round membranes on Si crystals were fabricated. The structure of the layers is shown in Fig. 1. The aluminum film was formed by the magnetron method from an Al-Si target, and the SiO2 layer was formed by the PECVD method. The radius of the membranes was 0.5 mm.

IV. Method of Experimental Determination of the Deformation of Membrane

The methods for measuring the mechanical characteristics are based on various optical methods for determining the deflection of membranes depending on the effects applied to them.

The measurement setup used in this work for studying the dependence of the deflection of membranes on the applied overpressure is described in detail in [14]. The main part of this setup is the Veeco Wyko NT 9300 optical profilometer, through which the deflection value was determined in the vertical scanning interferometry mode (VSI). An example of the measurement test is shown in Fig. 2. Due to the large slope of the structure, most of the points corresponding to the coordinates of the membrane surface are absent (empty areas can be seen on the surface profile), which is related to the features of the measurement method used, but this does not affect the final determination of the maximum deflection value.

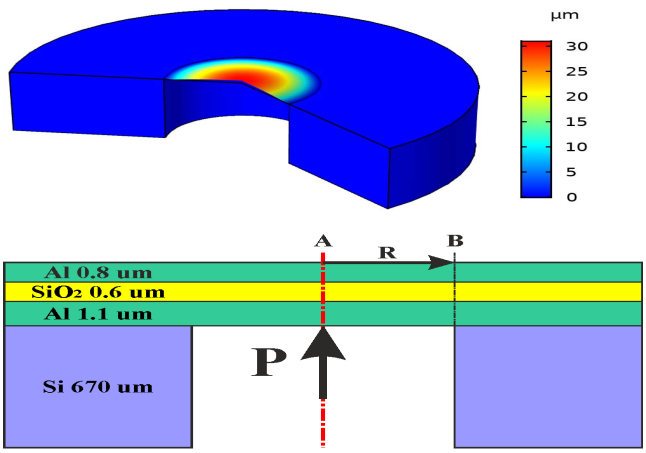

Fig. 1 A schematic view of the round membrane Al/SiO2/Al and its deflection under the applied pressure obtained in COMSOL Multiphysics software [15], where A-B is the line along with the corresponding mechanical stresses $\sigma_R, \sigma_T, \sigma_\mu$ were calculated.

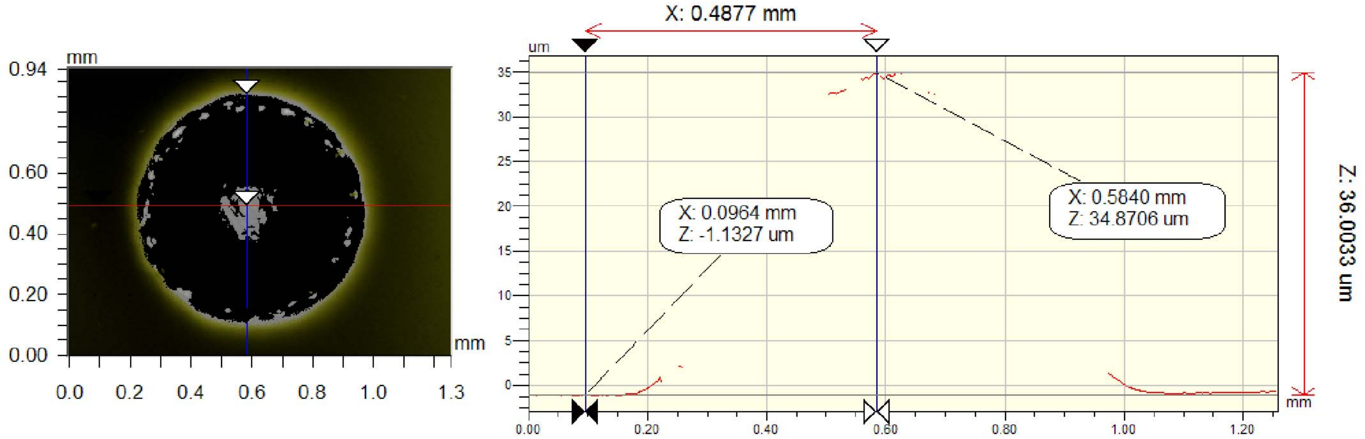

Fig. 2 Image of the membrane and its surface profile obtained during the experimental determination of the deflection

V. Numerical Modeling for Calculation of the Membran Bend

A. ANSYS software packages

The calculation was performed using the 8-node element Plane 183 with an axisymmetric option. The total number of mesh elements is 11770. The calculation was performed using geometric nonlinearity mode. Material models were taken linear and isotropic.

B. COMSOL Multiphysics software package

Three-dimensional axisymmetric structures was described in a two-dimensional workspace. The following boundary conditions were determined: the entire membrane together with the substrate is fixed along the lateral faces; overpressure was applied to the lower face of the membrane, as shown in Fig.1. The deformation of the studied membrane was determined using parametric modeling, where the overpressure in the range from 0 to 0.3 MPa was used as a parameter

VI. Results and Discission

In the process of numerical analysis of mechanical stresses in a round membrane, the following layer parameters were initially taken: averaged Young's modulus $E = 72$ GPa, membrane thickness $h = 2.5$ µm, membrane radius $a = 0.5$ mm, averaged Poisson's ratio $\mu = 0.3$ (since $\mu(\text{Al}) = 0.34$, $\mu(\text{SiO}_2) = 0.2$).

The dependences of the deflection and mechanical stresses (radial, tangential, and equivalent von Mises stresses) of an Al/SiO2/Al round membrane on the applied overpressure, obtained by simulation results in COMSOL Multiphysics, ANSYS [16], and also using the formulas (6-9) respectively is presented in Figures 3 and 4. It follows from the figures that the computations in two software packages (COMSOL Multiphysics, ANSYS) turn out to be almost identical and correspond to the calculation according to formulas (6-9).

The result of the calculation according to formula (5) at $C_2 = 8/3$ [17], performed under the assumption that there are no bending deformations (characterizing the first term in formula (5)) and no significant initial internal membrane stresses (characterizing the second term in formula (5)) differs by a two-fold difference with the data in Fig. 3. This effect may be connected with inaccurate representation or to the value of the Young's modulus or coefficient C_2, which for this structure may differ from the above values. When calculated by formulas (1, 2), the magnitude of the deflection and stresses are orders of magnitude higher than the data in Figure 3, which shows the inconsistency of using these calculation formulas for the analysis of membranes with significant deflection. This is consistent with the analysis given in [4, 9].

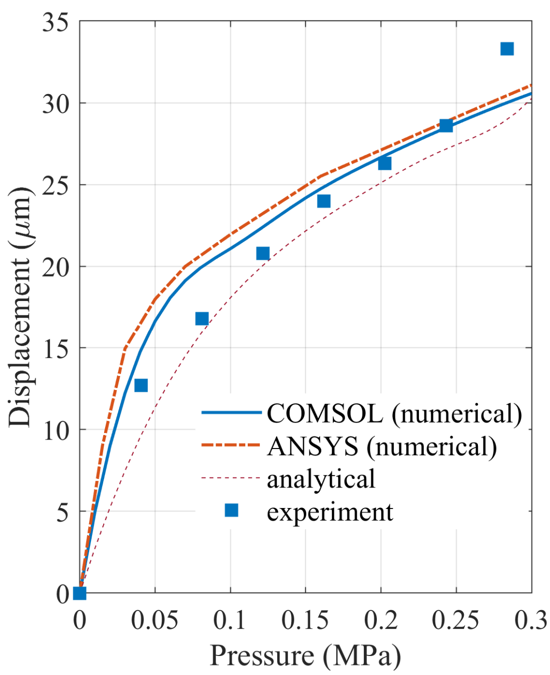

Fig. 3 Comparison of the magnitude of the deflection obtained in ANSYS, COMSOL Multiphysics, according to formula (9), obtained experimentally on a profilometer.

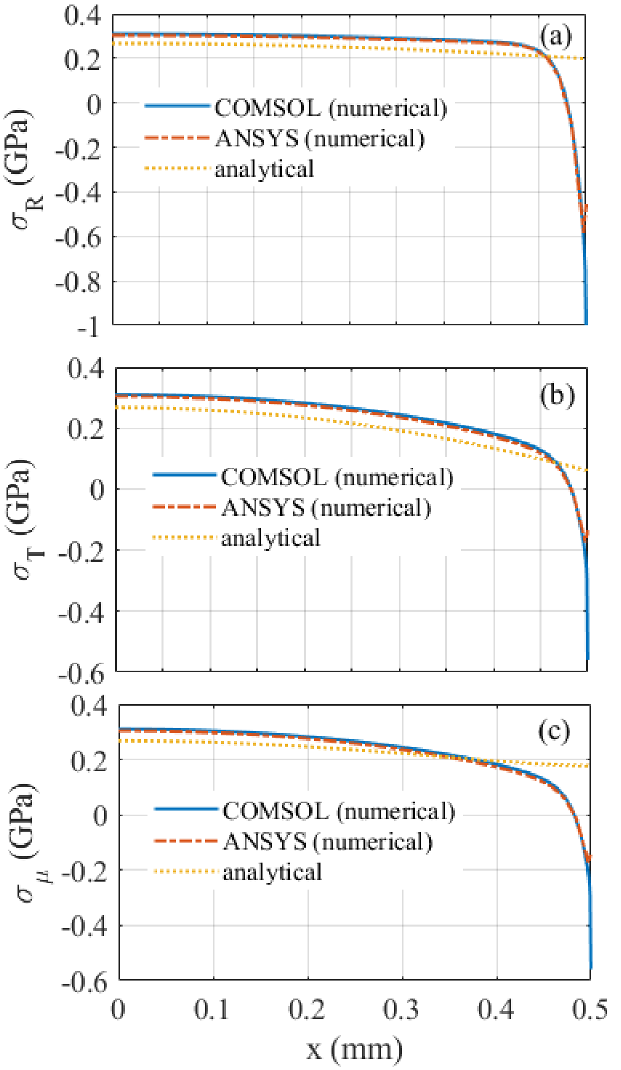

Fig. 4 Comparison of the values of mechanical stresses along the membrane surface σ_R (a), σ_T (b) и σ_μ (c) obtained in ANSYS, COMSOL Multiphysics and according to formulas (6-7).

Thus, it is shown that the optimal calculation procedure is to use the relations (6-9). However, one can see a discrepancy in the values near the fixing region (closed contour) on Fig.4.

Fig. 5 shows the results of calculating the value of mechanical stresses in COMSOL Multiphysics for the upper and lower surface of the membrane. Fig. 6 shows the distribution of mechanical equivalent stresses obtained in the ANSYS software package.

The above discrepancy between the calculation in software packages and the analytical relations is due to the fact that in the region of stress concentration, the stress state can no longer be considered flat. Also, when deriving analytical expressions for the bending of a circular membrane, the Saint-Venant principle applies [18]. In numerical modeling we can observe the stress distribution at any point in the membrane.

The stresses in the fixing region are critical for the membrane. The critical deformation for such membranes is the complete separation of the membrane from the Si crystal. Therefore, a detailed analysis of the stresses in the fixing region is a particular interest. This will be the subject of further research.

VII. Conclusions

An optical method was used to experimentally determine the deflection of a round Al/SiO2/Al membrane with a radius of 0.5 mm and a thickness of 2.5 µm, depending on the applied overpressure in the range up to 0.3 MPa. The magnitude of the deflection caused by the applied pressure of 0.3 GPa was more than 30 µm.

The dependence of the deflection of the membrane on the applied excess pressure is obtained. The distribution of stresses in the radial and tangential directions, as well as equivalent stresses (von Mises stresses) along the membrane surface are obtained. The highest value of von Mises stresses was obtained in the area of the fixed membrane boundaries.

The methods of calculating the dependence of the deflection of the membrane on the applied overpressure are compared. It was shown that in the study of the deflection of membranes, which significantly exceeds the thickness of the membranes, analytical relations related to formulas (6-9) should be used, which are obtained taking into account the presence of tensile forces, radial displacement, and elongation of the median plane.

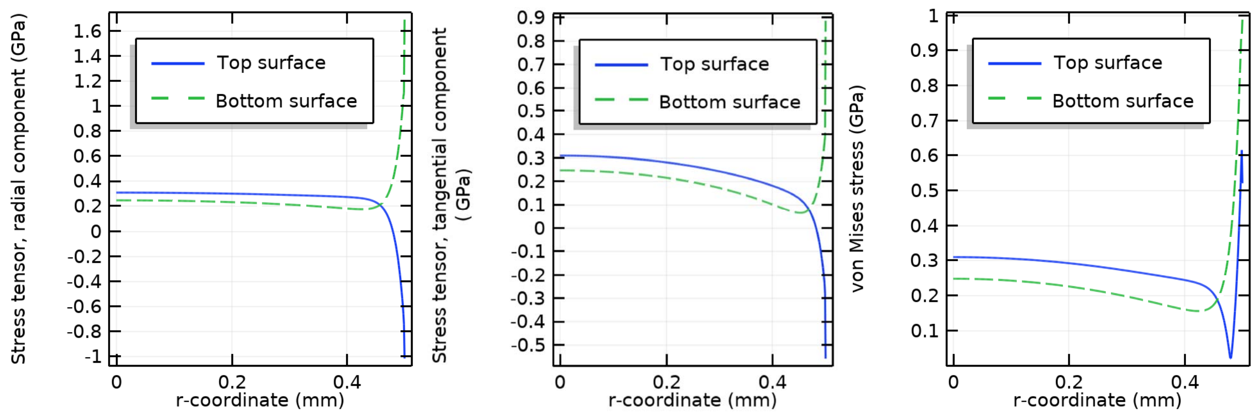

Fig. 5 Distribution of mechanical stresses σ_R, σ_T и σ_μ for the upper and lower surface of the membrane (COMSOL Multiphysics)

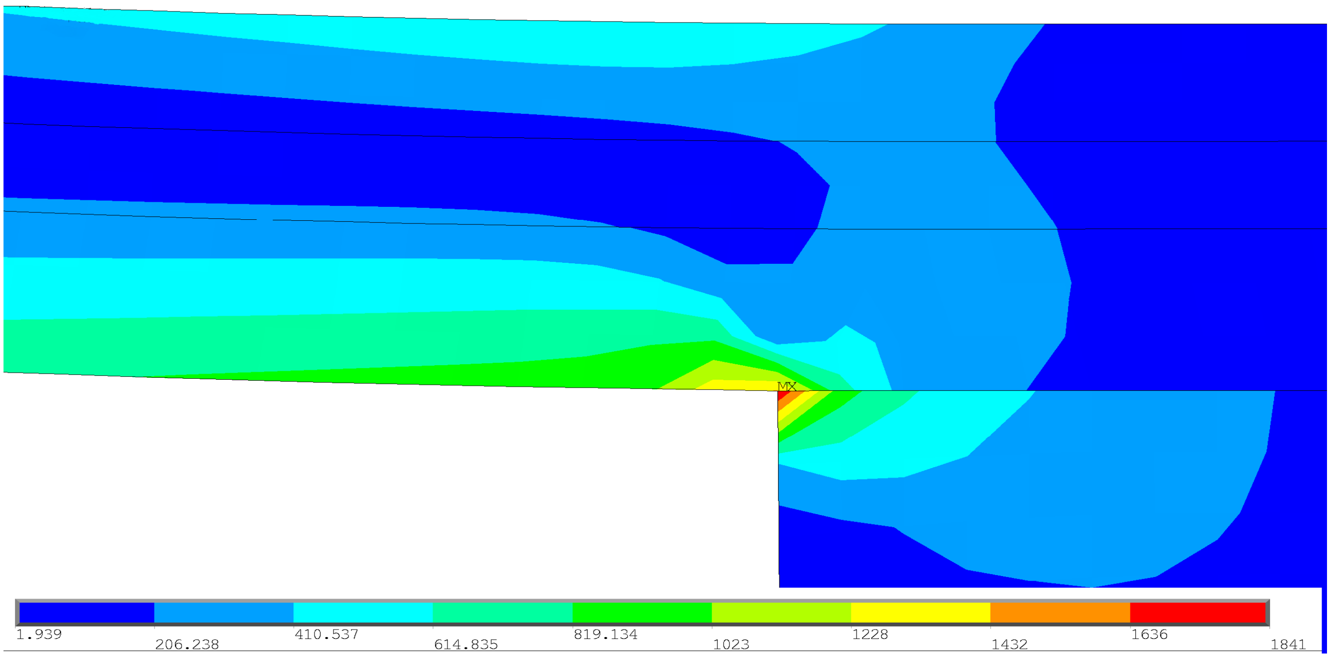

Fig. 6 Distribution of mechanical equivalent stresses (MPa) near the membrane fixing region (ANSYS)

The comparability of such an analytical calculation with the experimentally obtained values of the deflections and with the results of numerical modeling performed using various software systems (COMSOL Multiphysics, ANSYS) is shown. A discrepancy between the calculation in software packages and the analytical relations is due to the fact that this analytical relations does not take into account the features of fixing and geometry around the transition to embedment.

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